Scaling Analysis as a Pedagogical Tool in Teaching Transport & Reaction Processes

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1 – OVERVIEW OF POSTER

- Description of scaling analysis
- Value of scaling analysis
- 8-step methodology for implementing scaling analysis
- Illustrative example involving engineering data analysis
- Use in teaching fluid mechanics
- Use in teaching heat transfer
- Use in teaching mass transfer
- Use in teaching reactor design
- Use in teaching process design
- Student Feedback
- Summary
- Bibliography
- Acknowledgments

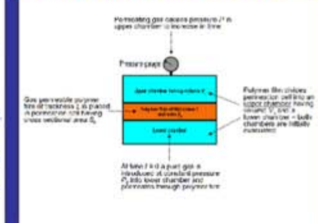
2 – DESCRIPTION & VALUE OF SCALING ANALYSIS

- Scaling analysis is a systematic method for nondimensionalizing a system of describing equations for a process so that all dimensionless variables and their derivatives have a magnitude ~1.
- This permits assessing the importance of various terms based on the values of the dimensionless groups that multiply them.
- As such, scaling analysis is an invaluable tool for educators since it provides a systematic way to arrive at model approximations.
- It thereby permits presenting disparate topics in transport and reaction processes in a unified and integrated manner.

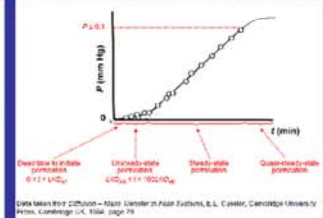
3 – EIGHT-STEP METHODOLOGY FOR SCALING ANALYSIS

1. Write the dimensional describing equations including any initial, boundary, and auxiliary conditions for the transport or reaction process being considered.
2. Define unspecified scale factors for the dependent and independent variables as well as appropriate derivatives appearing explicitly in the describing equations.
3. Define unspecified reference factors for each dependent and independent variable that is not naturally referenced to zero.
4. Form dimensionless variables by introducing the unspecified scale and reference factors for the dependent/independent variables and appropriate derivatives.
5. Introduce these dimensionless variables into the describing equations.
6. Divide through by the dimensional coefficient of one term in each equation, preferably a term that must be retained.
7. Determine the scale and reference factors by insuring that the principal terms in the describing equations are bounded between zero and order one [O(1)].
8. Use the resulting minimum parametric representation of the problem to explore the properties of the describing equations, how they can be simplified, and how useful information can be extracted from performance data for the process.

4 – EXAMPLE PROBLEM: MEMBRANE PERMEATION CELL



5 – PERFORMANCE DATA FOR PERMEATION CELL



Data taken from Diffusion – Mass Transfer in Fluid Systems, E.L. Cussler, Cambridge University Press, Cambridge U.K. 1984, page 79

6 – STEP 1: WRITE DESCRIBING EQUATIONS

$\frac{\partial \rho_A}{\partial t} = D_{AB}\frac{\partial^2 \rho_A}{\partial x^2}$ species balance in membrane

$\rho_A = 0,\ P = P_i$ at $t = 0$ chambers initially evacuated; P_i maintained in lower chamber

$\rho_A = HP_i$ at $x = 0$ equilibrium in lower chamber

$\rho_A = HP$ at $x = L$ equilibrium in upper chamber

$\frac{d}{dt}(\rho V_u) = \frac{V_u}{RT}\frac{dP}{dt} = -\frac{D_{AB}S}{M_A}\frac{\partial \rho_A}{\partial x}\Big|_{x=L}$ mass balance in upper chamber

DEFINITION OF SYMBOLS

- molar density
- diffusion coefficient of permeating component in membrane
- Henry's law gas-membrane equilibrium constant
- thickness of membrane
- molecular weight of permeating gas
- pressure
- gas constant
- cross-sectional area of membrane
- absolute temperature
- permeation time
- volume of upper chamber
- spatial coordinate measured from high pressure interface
- mass concentration of permeating component

7 – STEPS 2-7: INTRODUCE & DETERMINE SCALE FACTORS

$\rho_A^* = \frac{\rho_A}{\rho_s};\quad P^* = \frac{P}{P_s};\quad \left(\frac{dP}{dt}\right)^* = \frac{1}{P_s}\frac{dP}{dt};\quad x^* = \frac{x}{x_s};\quad t^* = \frac{t}{t_s}$

$\frac{x_s^2}{D_{AB}t_s}\frac{\partial \rho_A^*}{\partial t^*} = \frac{\partial^2 \rho_A^*}{\partial x^{*2}}$

$\rho_A^* = 0$ at $t^* = 0$

$\rho_A^* = \frac{HP_i}{\rho_s}$ at $x^* = 0$ — gives concentration scale

$\rho_A^* = \frac{HP_s}{\rho_s}P^*$ at $x^* = \frac{L}{x_s}$ — gives length scale

$\left(\frac{dP}{dt}\right)^* = -\frac{D_{AB}SRT\rho_s}{M_A V_u x_s P_s}\frac{\partial \rho_A^*}{\partial x^*}\Big|$ — gives pressure derivative scale

$P^* = \frac{P_i}{P_s}$ at $t^* = 0$ — gives pressure scale

time scale is the observation time t_o (i.e., any fixed time)

8 – STEP 8: ASSESS PROCESS BEHAVIOR

$\frac{1}{Fo_m}\frac{\partial \rho_A^*}{\partial t^*} = \frac{\partial^2 \rho_A^*}{\partial x^{*2}}$ where $Fo_m = \frac{t_o D_{AB}}{L^2}$

$\rho_A^* = 0,\ P^* = 1$ at $t^* = 0$

$\rho_A^* = 1$ at $x^* = 0$

$\rho_A^* = P^*$ at $x^* = 1$

$\left(\frac{dP}{dt}\right)^* = -\frac{\partial \rho_A^*}{\partial x^*}\Big|_{x^*=1}$

- $Fo_m \sim 1$ gives dead time to initiate permeation
- $Fo_m \sim 100$ gives time for onset of steady-state permeation
- $Fo_m > 100$, $P^* \ll 1$ steady-state permeation
- $P^* \approx 0.1$ gives pressure for onset of quasi-steady-state permeation

9 – SCALING ANALYSIS IN TEACHING FLUID MECHANICS

- Introduced after deriving equations of motion
- Used to assess applicability of fully developed flow
- Used to introduce creeping & lubrication-flow approximations
- Used to introduce hydrodynamic boundary-layer theory
- Used to assess importance of entrance, exit, and sidewall effects
- Used to assess quasi-steady-state flow approximation
- Used to assess possible approximations for free surface flows
- Used to assess compressible flow effects
- Used as alternative to Pi theorem for dimensional analysis

10 – SCALING ANALYSIS IN TEACHING HEAT TRANSFER

- Introduced after deriving energy equation
- Used to assess simplifications in multi-dimensional conduction
- Used to introduce film- & penetration-theory approximations
- Used to assess low Peclet number approximation
- Used to introduce thermal boundary-layer approximation
- Used to introduce low & high Biot number approximations
- Used to model heat transfer with phase change
- Used to address temperature-dependent physical properties
- Used to introduce Boussinesq approximation in free convection
- Used as alternative to Pi theorem for dimensional analysis

11 – SCALING ANALYSIS IN TEACHING MASS TRANSFER

- Introduced after deriving species-balance equation
- Used to assess bulk flow contribution to Fick's law
- Used to introduce film- & penetration-theory approximations
- Used to assess low Peclet number approximation
- Used to introduce solutal boundary-layer approximation
- Used to assess low & high Damköhler number approximations
- Used to assess low & high Thiele modulus approximations
- Used to model moving boundary mass-transfer problems
- Used to address concentration-dependent physical properties
- Used to introduce concept of Taylor dispersion
- Used to introduce concept of uniformly accessible surface
- Used as alternative to Pi theorem for dimensional analysis

12 – SCALING ANALYSIS IN TEACHING REACTOR DESIGN

- Used to introduce microscale/macroscale modeling
- Used to introduce concept of reaction boundary layer
- Used to identify slow, fast, & instantaneous reaction regimes
- Used to identify kinetic & diffusion domains
- Used to assess quasi-stationary hypothesis
- Used to develop design precepts for chemical reactors
- Used to develop models for mass-transfer coefficients
- Used to design continuous stirred tank reactor
- Used to design co- and countercurrent flow reactors

13 – SCALING ANALYSIS IN TEACHING PROCESS DESIGN

- Design of a compact membrane-lung oxygenator for chronic obstructive pulmonary disease patients[3]
- Design of rapid cycle pressure-swing adsorption process for the production of oxygen-enriched air for synfuels production[7]
- Design of the thermally induced phase-separation process for fabricating high porosity polymeric membranes[9]
- Design of the fluid-wall aerosol flow reactor for producing hydrogen fuel in the absence of any greenhouse gas emissions[8]

(Superscript numbers refer to references in bibliography)

14 – STUDENT FEEDBACK

CN5391 – Scaling Analysis in Modeling Transport & Reaction Processes Semester II – 2006

WBK Score for Teaching Effectiveness:	4.86/5.00
CHBE Department Average Faculty Score:	3.79/5.00
NUS Average Faculty Score:	3.88/5.00

Selected Student Comments

- Extremely effective teaching methods; created immense interest in the field; highly motivational; transport phenomena is no longer a pain now!
- A very cheerful and helpful teacher who is able to describe out what is happening in the system and why are we scaling the system.
- Please find a way to teach a graduate course in transport phenomena or thermodynamics in your next visit to Singapore. Sincere appreciation for the extremely interesting course on scaling.
- His class is always interactive since he facilitates discussions very well. Through these discussions, students are not only learning from classroom instructions, but our analytical and critical thinking to respond are also being enhanced. Moreover, as an educator, he manages to present difficult subjects through direct, great, and interesting analogies.
- His aim is help students to grasp the skills of this module in order to solve problems…he helps students to recognize the beauty of the method.

15 – SUMMARY

- O(1) scaling analysis provides a systematic method for nondimensionalizing a system of describing equations that is distinct from conventional dimensional analysis
- O(1) scaling analysis is a valuable pedagogical tool for introducing the classical approximations made in teaching transport and reaction processes
- O(1) scaling analysis can be used as a systematic method for honing student skills in the art of approximation
- O(1) scaling analysis can be integrated into teaching courses in fluid mechanics, heat transfer, mass transfer, reactor design, process design, and mathematical modeling

16 – BIBLIOGRAPHY

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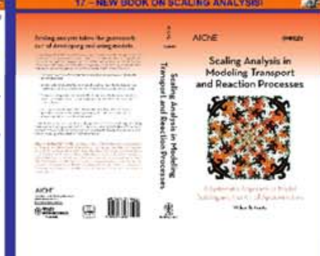
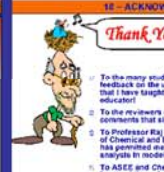
17 – NEW BOOK ON SCALING ANALYSIS!



18 – ACKNOWLEDGMENTS



- To the many students who have provided constructive feedback on the use of scaling analysis in the courses that I have taught during my 39 years as an engineering educator!
- To the reviewers of this paper for their constructive comments that significantly improved this presentation.
- To Professor Raj Rajagopalan, Head of the Department of Chemical and Biomolecular Engineering at NUS, who has permitted me to teach an annual course on scaling analysis in modeling transport and reaction processes.
- To ASEE and Chemical Engineering Education Journal for providing a forum for me to present my ideas on using scaling analysis as a pedagogical tool in teaching transport and reaction processes.
- To John Wiley and Sons Publishers for giving me an opportunity to publish a book that hopefully will be a definitive treatment of the use of scaling analysis in engineering education.